Mental Math Cheat Sheet

Two tricks that grow out of squaring numbers ending in 5

1 Same tens digit, units digits that add to 10

$$43 \times 47 = \underbrace{20}_{4 \times 5}\underbrace{21}_{3 \times 7}$$

When does it work?

- The tens digits are the same: 43 and 47 both have 4 tens.
- The units digits add up to 10: 3 + 7 = 10.

Other examples: 62 × 68, 84 × 86, 71 × 79.
But this doesn't work for 43 × 48 (units add to 11) or 43 × 57 (tens differ).

The steps

1. **Front digits.** Multiply the tens digit by the next number up: 4 × 5 = **20** (your front digits).
2. **Last two digits.** Multiply the two units digits: 3 × 7 = **21** (your last two digits).
3. **Join them.** Write the front digits, then the last two digits, side by side: **20** and **21** make **2021**.

Watch out. The last two digits always take two places. If the units product is a single digit, put a 0 in front of it. For 41 × 49: 4 × 5 = 20 and 1 × 9 = 09, so the answer is 2009.

One more example: 27 × 23

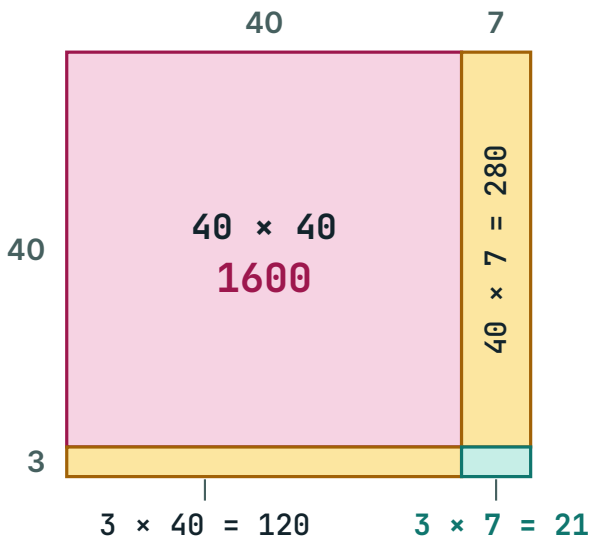
The tens digits match (2 and 2) and the units add to 10 (7 + 3).

Front digits: 2 × 3 = **6**. **Last two digits:** 7 × 3 = **21**.

Join them: **6** and **21** make **621**. The front digits can be a single digit, as they are here.

Quick check: 27 × 23 is close to 25 × 25 = 625, and 621 is close to that.

Why it works: Trick 1



Split each number into tens and units: 43 = 40 + 3 and 47 = 40 + 7. Multiplying gives four pieces, as in the picture.

The big square is 40 × 40 = 1600.

The two strips are 40 × 7 = 280 and 3 × 40 = 120. Because 7 + 3 = 10, together they make 40 × 10 = 400, the same as one strip of 40 by 10.

The square and strips together make 1600 + 400 = 40 × 50 = 2000. Since 40 × 50 = (4 × 5) × 100, that is 20 hundreds: your front digits.

The small corner is 3 × 7 = 21, your last two digits.

What if the units digits don't add to 10? Try 43 × 48. The trick would give 4 × 5 = 20 and 3 × 8 = 24, so 2024. But the real answer is 2064. The two strips are now 40 × 8 = 320 and 3 × 40 = 120, which make 440, not 400. That extra 40 is why the units digits must add to exactly 10.

In algebra. Let the tens digit be t, and the units digits a and b with a + b = 10. Then $(10t + a)(10t + b) = 100t^2 + 10t(a + b) + ab = 100t^2 + 100t + ab = 100 \times t(t + 1) + ab$. The part t(t + 1) lands in the hundreds, and ab fills the last two digits.

Try it yourself

Answers are on the last page.

62 × 68 **84 × 86** **71 × 79**

2 Two numbers equally far from a round number

$$47 \times 53 = (50 - 3)(50 + 3) = 50^2 - 3^2$$
$$= 2491$$

50 is the middle, **3** is the distance

When does it work?

- Find the middle: the number exactly halfway between the two. For 47 and 53, the middle is 50.
- Both numbers are the same distance from it: 47 is 3 below 50, and 53 is 3 above.

Other examples: 48 × 52 (middle 50), 96 × 104 (middle 100), 63 × 67 (middle 65).
But this doesn't work for 47 × 54.

The steps

1. **Find the middle and the distance.** Add the two numbers and halve: (47 + 53) ÷ 2 = **50**. The distance is 53 − 50 = **3**.
2. **Square the middle.** 50 × 50 = **2500**. If the middle ends in 5, use the 5s trick.
3. **Square the distance and subtract it.** 3 × 3 = **9**, so 2500 − 9 = **2491**.

Watch out. You subtract, you never add. The answer is always a little less than the middle squared, which gives a quick check: 47 × 53 must be just under 2500. The middle is easiest to square when it ends in 0 or 5. For 63 × 67 the middle is 65, so use the 5s trick: $65^2 = 4225$, then $4225 - 2^2 = 4221$.

One more example: 58 × 62

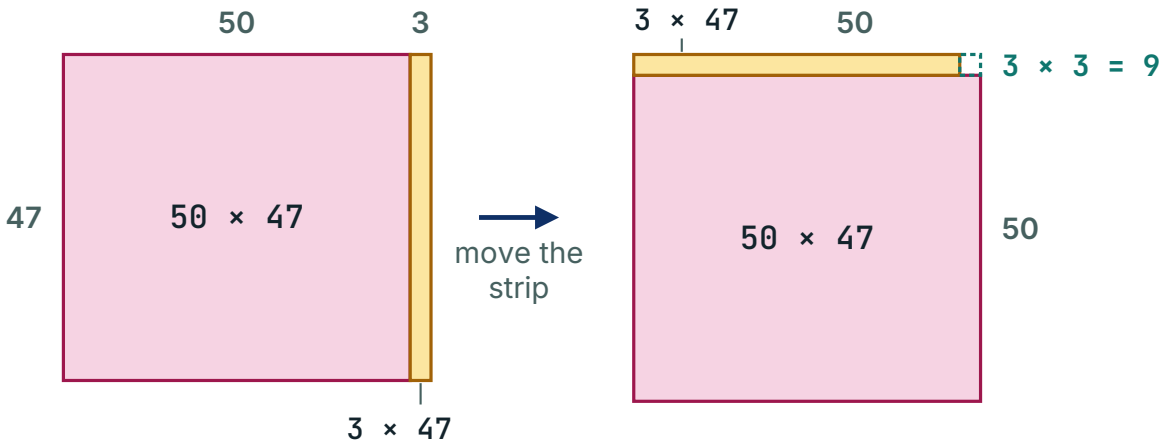
The middle is 60, and both numbers are 2 away from it.

Square the middle: 60 × 60 = **3600**. **Square the distance:** 2 × 2 = **4**.

Subtract: 3600 − 4 = **3596**.

Quick check: 58 × 62 should be just under 3600, and 3596 is.

Why it works: Trick 2



The rectangle is 53 wide and 47 tall, because 47 = 50 − 3 and 53 = 50 + 3. Cut a strip 3 wide off the right side and turn it to lie along the top. The pieces now fill a 50 × 50 square, except for a small 3 × 3 corner that is missing. So the area is 50 × 50 − 3 × 3 = 2500 − 9 = 2491.

What if the distances are different? Try 47 × 54. Now 47 is 3 below 50 but 54 is 4 above, so the strip you cut off no longer fits neatly and the trick gives the wrong answer. The real answer is 2538.

In algebra. With middle m and distance d, the two numbers are m − d and m + d, and $(m - d)(m + d) = m^2 - d^2$. This is the difference of squares identity, read backwards (here m = 50 and d = 3).

Try it yourself

The last one uses the 5s trick.

48 × 52

96 × 104

57 × 63

63 × 67

Check your answers. Trick 1: 62 × 68 = 4216, 84 × 86 = 7224, 71 × 79 = 5609. Trick 2: 48 × 52 = 2496, 96 × 104 = 9984, 57 × 63 = 3591, 63 × 67 = 4221.

Which trick surprised you most? Hit reply on the welcome email and tell me.